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RESEARCHES REGARDING THE DEVELOPMENT OF A MATHEMATICAL MODEL TO OPTIMIZE THE OPERATION OF THE ANTI-LOCK BRAKING SYSTEM

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Abstract—This paper presents a simplified model of a passenger-car's ABS (Anti-Lock Braking System). For simplicity purpose, a "quarter-car" planar model was considered: a wheel in rotation movement and a mass in translational movement (the vehicle mass supported by that wheel). In these conditions, a very simple dynamic model was obtained, having only two degrees of freedom (DOFs): the wheel rotation and the translation of the "quarter-car" mass. The mathematical model consists in the motion equations of the wheel and "quarter-car" mass and in the equations describing the comportment of the driver, controller, hydraulic module and brake. The model was transposed in a Matlab-Simulink model, offering the possibility to simulate many times how the ABS works if the constructive parameters, road conditions or driver inputs are changed.

Keywords—ABS, vehicle dynamics, Matlab-Simulink, model, simulation

I. INTRODUCTION

TO check the ABS working algorithm and to emphasize the different influences on the braking process, with and without ABS, a planar dynamic model for a single wheel of the vehicle was conceived and used. The selected model has two inertial elements: a wheel in rotation and a mass in translational movement (the vehicle).

The influences of the suspension and steering mechanisms were not explicitly considerate [1]. Also were neglected the jerk and jumping movements of the wheel, respectively the lift-up and pitch movements of the translational mass. These simplifications were realized deliberately to highlight the anti-lock braking system parameters that influence the braking process. In these conditions, the dynamic model has only two degrees of freedom: the wheel rotation around the spindle and the translation of the mass sustained by this wheel (including the wheel mass) on the travelling (longitudinal) direction [2].

The wheels locking during braking is unwanted for few very important reasons [3]: the loss of the tire ability to generate lateral grip force; the diminution of the grip force (which will lead to a smaller braking deceleration); the intense wear of the tire. In order to eliminate these negative effects, the aim of the ABS system is to avoid the locking of the wheels during braking. If the driver pushes to hard the braking pedal, the ABS controller will try to maintain the most preferable slip ratio of the wheel, the one permitting to the tire to generate the maximal grip force.

Another problem of the ABS systems is the harsh noise and the pedal kickback. Due to these facts, the driver might have problems in controlling the vehicle. This is the reason why recent trends of research and development for vehicular ABS are focused on the NVH (Noise, Vibrations, Harshness) as well as braking performance. Brake experts gradually recognized that continuous flow/pressure control, which eliminates abrupt pressure change, is the most efficient countermeasure for these problems. One type of system that enables continuous pressure rise to eliminate pulse-up noise and vibration of ABS is LFC (Linear Flow Control) [4].

The ABS braking cycle of conventional ABS systems available on the market is divided into different phases, hereby trying to keep the slip of the wheels at the optimum wheel slip with maximum tire traction as long as possible. Good systems reach an efficiency of η_{ABS}=0.95. However, an ideal ABS would be able to keep the wheel slip at a point of maximum tire traction and reduce the braking distance to a minimum [5].

The state-of-the-art in ABS control system development requires stand-alone desktop simulation capability, HIL (Hardware-In-The-Loop) simulation capability, and real vehicle testing. The typical HIL system for a brake system might include some or all of the production brake components in real form, yet the vehicle body, drive train, and suspension, road profile, and driver themselves are modeled in some mathematical fashion [6].

In this paper, it is presented a Simulink model of ABS, which was built using the recent researches made in the ABS field.

II. THE SIMULINK MODEL OF ABS

Simulink is a work module of the general mathematical calculus program Matlab (Mathworks Inc.). It is suitable for the simulation of technical systems because it has some very valuable qualities [7]:

- It allows the description of the modeled system-using block diagrams interconnected with signal lines; from here results the ease in models creation, understanding and use.
- The block diagrams can be hierarchically grouped, which permits the description of some subassemblies and assemblies of high complexity.
- The writing of the algebraic and differential equations with initial values, that constitute the mathematical model of the system, is done automatically.
- For solving the mathematical model, more advanced methods of numerical integration are available more.
- The simulation's results can be represented graphically in different ways and obtained as a function of time.
- The results obtained for one simulation can be stored in Matlab clipboard or file memory, in order to realize further analysis and comparisons.

"Fig 1" presents the main diagram of the Simulink model, diagram realized for the simulation of the ABS operating mode. The blocks that compose it are defined to permit the identification and understanding of the ABS functioning.

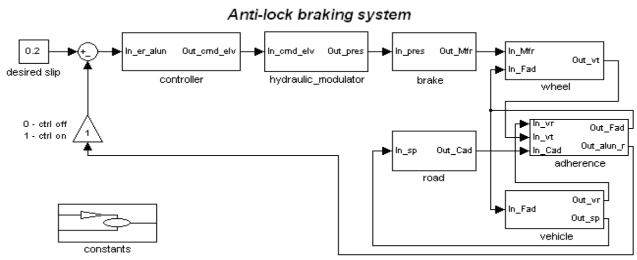

Fig. 1. Diagram of the Simulink model for ABS simulation

The "Wheel" and "Vehicle" blocks correspond to the two degrees of freedom of the dynamic model – wheel rotation and "quarter vehicle" translation. These two movements are connected through the wheel-road grip (the "Adherence" block), that depends on the road properties defined in the "Road" block.

The comparator (the round block from the upper-left side of the "Fig 1") calculates the difference between the desired values of the slip (the "Desired Slip" block) and the real slip (calculated in the "Adherence" block). Using this difference, the "Controller" block, that simulates the operation mode of the adjustment algorithm implemented in the ABS computer, will establish what kind of command must be executed by the ABS hydraulic assembly (modeled by the "Hydraulic Modulator" block). The pressure generated by this is finally applied to the brake (modeled by the "Brake" block).

"Ctrl off / Ctrl on" block was introduced in the inverse connection path (to enable or disable the feedback loop) in order to simulate with the same model both an anti-lock braking system (ABS) and a classic braking system (without ABS).

The "Constants" block, presented in "Fig 2", defines the main parameters of the simulation: the mass, the moment of inertia of the wheel J, the dynamic radius r_d of the wheel and the initial vehicle speed v_0.

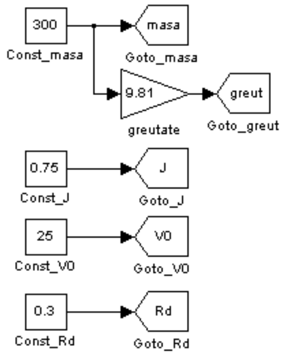

Fig. 2. "Constants" block

The "Road" block permits the assessment of the way in which the grip coefficient changes as function of the distance travelled by the vehicle. In this case, presented in "Fig 3", the dependency grip coefficient – distance is defined with the help of a table, the passing from one value to another being realized using a linear interpolation.

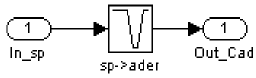

Fig. 3. "Road" block

Based on the wheel's real speed v_r and theoretical (peripheral) speed v_t, that are inputs for the "Adherence" block, "Fig 4", the braking slip coefficient is calculated.

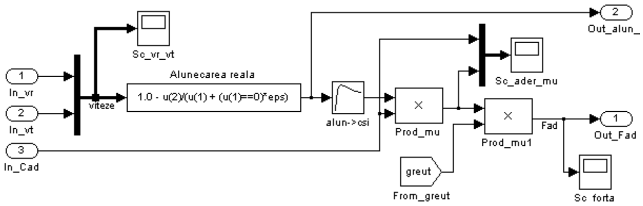

Fig. 4. "Adherence" block

This value (*alun_r*, which is also the second output of the block "Adherence") is introduced as input for the linear interpolation sub-block "alun->csi", that calculates the ratio (the fraction, with values between 0 and 1) of the current "used" grip and the maximum available grip. By multiplying this amount with the adherence coefficient C_{ad} (third input of the "Adherence" block) it obtains the coefficient of the "used" grip, *mu*. Finally, by multiplying *mu* with the tire load (considered equal with the weight supported by the wheel) it obtains the grip force used effectively F_{ad} (the first output of the "Adherence" block).

This value of the "used" grip becomes an input for the blocks "Wheel" ("Fig 5") and "Vehicle" ("Fig 6"), which calculate the dynamics of wheel and vehicle.

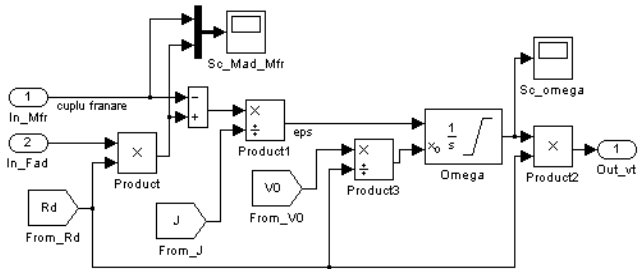

Fig. 5. "Wheel" block

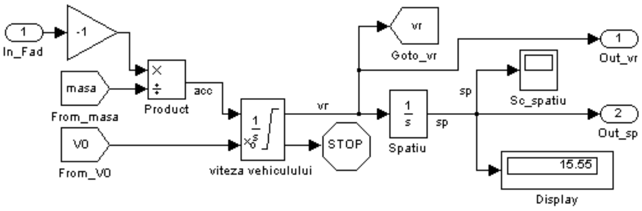

Fig. 6. "Vehicle" block

In the "Wheel" block, the grip force F_{ad} (from the second input) is multiplied with the dynamic radius r_d in order to calculate the grip moment. Then, the braking moment is subtracted from this, thus obtaining the sum of the exterior moments that act on the wheel. This is divided by the moment of inertia *J* in order to obtain the angular acceleration of the wheel ε, which is then integrated (with an initial value $\omega_0=v_0/r_d$ that corresponds to the slip absence) to obtain the angular speed ω. By multiplying it with the dynamic radius, it is calculated the theoretical speed v_t, that is the output of this block.

In the "Vehicle" block, the grip force F_{ad} will have an opposite orientation (multiplication with -1), because when braking, the interaction forces between the wheel and the road is resisting to the vehicle movement. Then, dividing the grip force by the vehicle mass (the mass supported by the analyzed wheel), it determines the braking acceleration (it has negative value). Starting from the initial value v_0 of the speed and integrating the vehicle's acceleration, it obtains the real speed of the vehicle v_r. When a very small speed of the vehicle is attained, the simulation will be stopped (the "STOP" block). A second integration, but this time the integration of the real speed, will give as a result the distance travelled by the vehicle, necessary to define the road properties.

The wheel-road slip coefficient is usually calculated by the ABS controller, using a specific algorithm and based on the information coming from the wheel sensors. Starting from the angular speed of the wheel obtained from its sensor and from the estimated wheel radius, the peripheral (tangential) speed v_t is calculated first. Then, based on the peripheral speeds of all the wheels, the real translational speed of the vehicle (in a control point, for example, the centre of gravity) and its yawing speed are estimated [8]. With these translational and yawing speeds can be deduced the real speeds v_r of all the wheels and, finally, the slip ratio of each wheel, with the relation bellow:

$$s = 1 - v_t / v_r \tag{1}$$

In the presented case, the slip coefficient, calculated by the "Adherence" block, is transmitted as a reaction value to a comparator (as can be seen on the "Fig 1"). By multiplying the slip with 0 ("Amplification" block) any tendency to block the wheel will be neglected and the model will correspond to a classical braking system without ABS. If the amplification block has the value 1, then the slip value will arrive to the comparator, and the model will correspond to an ABS braking system.

By subtracting the real value from the desired value of the slip ratio, the comparator determines the actual control error (the difference between the optimum slip and the momentary slip). This error represents the input value for the "Controller" block. In the Simulink model, two types of controllers were used. These are presented in the "Fig 7" and "Fig 8".

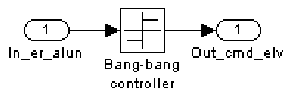

Fig. 7. "Controller" blocks with two states controller

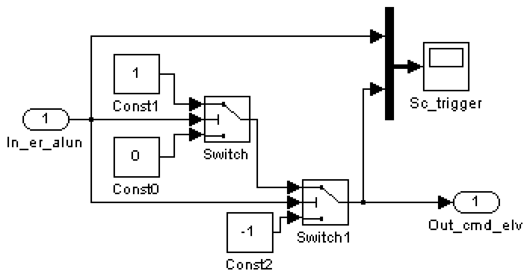

Fig. 8. "Controller" blocks with three states controller

The two states controller (also called "bang-bang" or "on-off" controller), the most simple possible, is presented on the "Fig 7". This is modeled with the mathematical function "signum" and its output will have one of the values -1 or 1 (theoretically, the value 0 is also possible, but in practice the probability to obtain it is very small, only when the desired slip is equal with the real slip). For the implementation, such a controller needs only a valve with two states (closed-open) in the hydraulic modulator, which will result in constructive simplicity and lower price.

The ABS functioning principle adopted here is the following [9]:

- If the real slip is smaller than the desired slip (the wheel is under-braked), the controller commands the valve in the position in which that permits to the braking fluid (pushed by a pump) to be introduced in the slave cylinder (that actuates the brake). Thus, the pressure increases, the wheel decelerates its spin and the slip increases.

• If the real slip is bigger than the desired slip (the wheel tends to lock), the controller commands the valve in order to permit the brake's cylinder to be purged (part of the liquid is eliminated from the cylinder). Consequently, the actuation pressure reduces, the brake weakens, the wheel accelerates and the slip decreases.

The three state controller works with two slip thresholds near the optimum value, one over and one under the optimum slip ratio. This may be implemented either with two valves with two states (closed-open), or with one valve with three states (closed-isolated-open). When the slip is between the two thresholds, the valves remain in the position "isolated" (without the need to increase or decrease the pressure, maintaining the pressure generated by the driver's actuation on the brake pedal). In the other two situations (when the slip exceed the superior threshold, or is under the inferior threshold), the controller with three states will operate as the two state controller, purging or filling the slave cylinder. In these conditions, the controller will generate three output values: -1, when the pressure in the cylinder must decrease, 0, when the pressure remains unmodified and 1, when the pressure must increase.

The command signal generated by the controller is applied to the "Hydraulic modulator" block, with the scheme presented in the "Fig 9". Its firs element is introduced in order to consider the delay between the moment when the brake action command is executed and the moment when the pressure reaches its effective value. This is modeled as a transfer function (on the scheme the delay is 0,002 s).

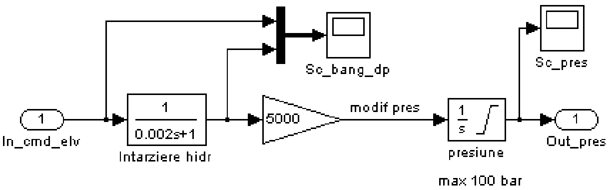

Fig. 9. "Hydraulic modulator" block

The next element, an amplifier, introduces the rate of the pressure increase reachable when the valve is completely open (on the scheme it is equal with 5000 bar/s). In addition, this block allows the control of the maximum rate of pressure increase, in order to consider the brake-pedal actuation-speed realized by the driver. The value obtained at the amplifier's output represents the pressure change per time unit (the time-related derivative of the actuation pressure).

The last element, the integration block, permits the calculation of the pressure applied to the piston of the (disk or drum) brake. This block ensures also the inferior limitation (the constant value 0, that indicate the absence of pressure) and superior limitation (that was modified during the simulation to consider the maximal pressure generated by the driver push on the brake pedal).

The last block of the Simulink model, the "Brake" block ("Fig 10"), simulates the way in which the actuation pressure is transformed in braking moment. The modality used is a simple one, by utilizing a mechanical amplifier. The value on the scheme (110 Nm/bar) was selected so that to correspond as well as possible to the functioning of the braking force distributor, i.e. to ensure the wheel lock on dry asphalt only for very high values of the actuation pressure.

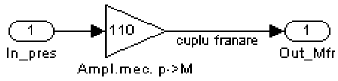

Fig. 10. "Brake" block

Even if it is simple, the model was realized in a systemic approach, intending to emphasize the functional connections between the different components of the ABS system. The model can be developed further by replacing a simple functional block with a more complex one (exactly how the two states controller was replaced with a three states controller), keeping the same general scheme and maintaining all the inputs and outputs. Also, it can be extended easily from a system with only one wheel, to a system with four wheels or more.

III. UTILIZATION OF THE ABS MODEL

The model presented in the previous chapter was runed 20 times, the input data being modified so that comparisons and evaluations of its influences to be possible. As it can be seen in the table bellow, the following parameters have been changed [9]:

• the ABS system is active or inactive (classical braking system);

• the controller with two or three states;

• the maximum braking pressure of 45, 55, 75 and 90 bar (results by pressing the brake pedal);

• the hydraulic delay of 0.005 and 0.01 s;

• the hydraulic gain of 5000 and 2000 bar/s;

• the mass distributed on a single wheel of 300 and 200 kg (including the mass of the wheel);

• the equivalent moment of inertia of the wheel 0.75 and 15 kg.m^2;

• the speed at the braking begin of 25 and 40 m/s (90 and 144 km/h);

• smooth road surface with constant adherence (dry asphalt μ=1, wet asphalt μ=0.55, snow μ=0.2, glazed frost μ=0.1); smooth road with variable adherence (dry asphalt μ=1, followed by a section of 5 m with snow μ=0.2 and back on dry asphalt); bumpy road with constant adherence (jumping on a bump of 30 mm on dry asphalt μ=1).

The other parameters of the simulation are:

• Three states controller (it commands the decrease, the maintaining or the increase of the pressure).

• The pressure in the hydraulic circuit equal with 75 bars.

• System delay equal with 0.005 s.

• Hydraulic gain 5000 bar/s.

- Mass on the wheel equal with 300 kg.
- The inertia moment of the wheel equal with 0.75 $kg.m^2$.
- The speed from which starts the braking equal with 25 m/s (90 km/h).
- The road surface used for realizing the simulation begins with a section of dry asphalt of 5 m, then it is switched to a surface with snow, with an adherence coefficient equal with 0.2 for 2 m distance and then return on dry asphalt.

The simulations carried out with an emphasis on the input parameters used are listed in the TABLE I and the results presented in the previous figures, correspond to the following situations: active ABS (“Fig 11”) and inactive ABS (classical braking system – “Fig 12”).

TABLE I
THE INPUT PARAMETERS USED IN THE SIMULATIONS

No.	Simulation name	ABS	Ctrl	Pressure [bar]	Hydr. Delay [s]	Gain [bar/s]	Mass [kg]	Inertia moment [$kg.m^2$]	v_0 [m/s]	Road
1	1ABS2_p90_t005on	1	2 st	90	0.005	5000	300	0.75	25	2 m snow
2	1ABS2_p90_t005on_	1	2 st	90	0.005	5000	300	0.75	25	2 m snow
3	1ABS3_p90_t005on	1		90	0.005	5000	300	0.75	25	2 m snow
4	1ABS3_p90_t005on_	1		90	0.005	5000	300	0.75	25	2 m snow
5	1ABS3_p90_t005on_15kgm2	1		90	0.005	5000	300	1.5	25	2 m snow
6	1ABS3_p90_t005on_200kg	1		90	0.005	5000	200	0.75	25	2 m snow
7	1ABS3_p90_t005on_drum	1		90	0.005	5000	300	0.75	25	5 m snow
8	1ABS3_p90_t005on_jos	1		90	0.005	5000	300	0.75	25	2 m snow
9	1ABS3_p90_t005on_polei	1		90	0.005	5000	300	0.75	25	glazed frost 0.1
10	1ABS3_p90_t005on_sus	1		90	0.005	5000	300	0.75	25	2 m snow
11	1ABS3_p90_t005on_ud	1		90	0.005	5000	300	0.75	25	wet asphalt 0.55
12	1ABS3_p90_t005on_uscat	1		90	0.005	5000	300	0.75	25	dry asphalt 1
13	1ABS3_p90_t005on_v40	1		90	0.005	5000	300	0.75	40	2 m snow
14	1ABS3_p90_t005on_zapada	1		90	0.005	5000	300	0.75	25	snow 0.2
15	2ABS3_p75_t005off	0		75	0.005	5000	300	0.75	25	2 m snow
16	2ABS3_p75_t005on	1		75	0.005	5000	300	0.75	25	2 m snow
17	3ABS3_p45_t005off_drum	0		45	0.005	5000	300	0.75	25	5 m snow
18	3ABS3_p55_t005off	0		55	0.005	5000	300	0.75	25	2 m snow
19	4ABS3_p90_t010on	1		90	0.01	5000	300	0.75	25	2 m snow
20	5ABS3_p90_t005on_gain2	1		90	0.005	2000	300	0.75	25	2 m snow

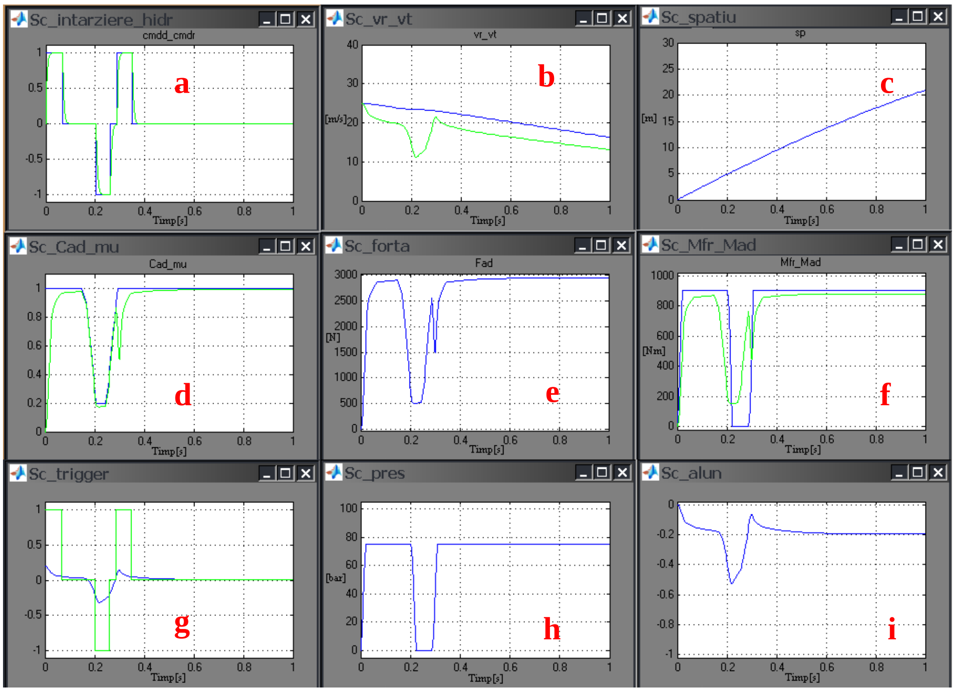

Fig. 11. Example of simulation results (with ABS): a – hydraulic delay (where it can be observed the difference between the solenoid valve command and its opening); b – the real speed and the theoretical speed; c – the distance traveled by the vehicle; d – the adherence coefficient of the road and the used adherence coefficient (it indicates the available value and how much has been used); e – the effective adherence force (used adherence, which represents the wheel contribution to the vehicle braking effort); f – the braking moment (which tends to lock the wheel) and the adherence moment (which tends to accelerate the wheel); g – the difference between the wanted slip ratio and the realized one and the solenoid valve command; h – the pressure in the hydraulic circuit; i – the effective slip of the wheel.

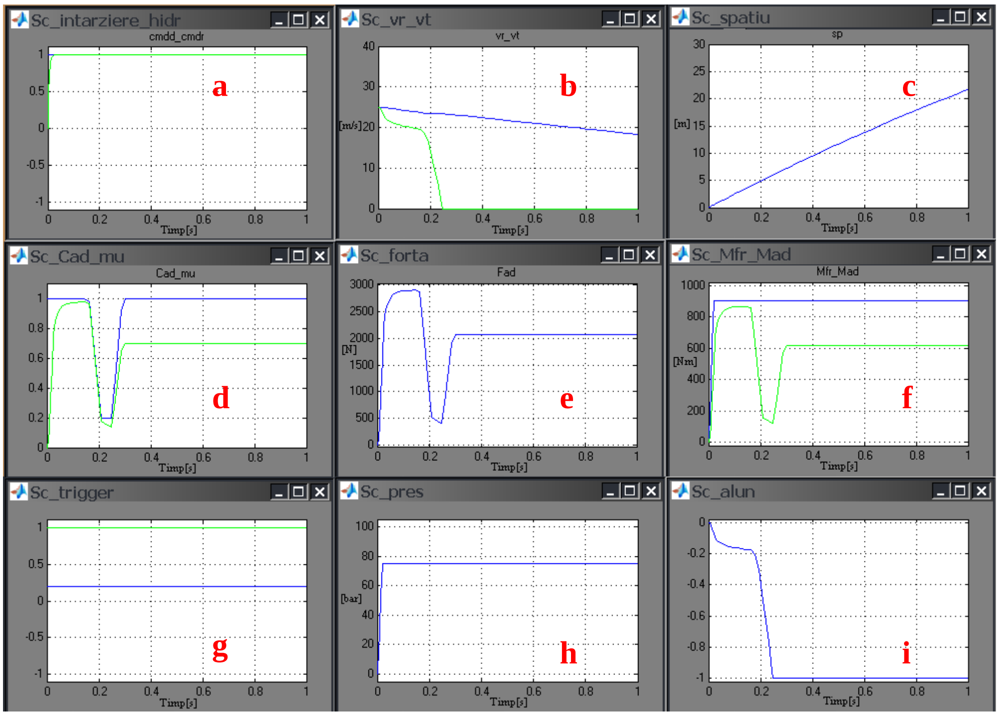

Fig. 12. Example of simulation results (without ABS): a – hydraulic delay (where it can be observed the difference between the solenoid valve command and its opening); b – the real speed and the theoretical speed; c – the distance traveled by the vehicle; d – the adherence coefficient of the road and the used adherence coefficient (it indicates the available value and how much has been used); e – the effective adherence force (used adherence, which represents the wheel contribution to the vehicle braking effort); f – the braking moment (which tends to lock the wheel) and the adherence moment (which tends to accelerate the wheel); g – the difference between the wanted slip ratio and the realized one and the solenoid valve command; h – the pressure in the hydraulic circuit; i – the effective slip of the wheel.

IV. Conclusion

Analyzing the two simulations, the first one with the ABS system active and the second one with the ABS system inactive, it can be observed a significant improvement of the braking effectiveness. Unlike, the first case in which the ABS system is active and prevents the wheel locking when the vehicle passes from asphalt to snow, when the ABS system is off, the wheels lock, loosing the vehicle control and the braking distance is increased.

The novelty of this research paper is represented by the definition of the simplified mathematical model of a passenger-car's ABS for one single wheel ("quarter-car") taking into consideration: a wheel in rotation movement and a mass in translational movement (the vehicle mass supported by that wheel). The author's personal contributions are determined by the transposition of this simplified mathematical model into a new model Matlab-Simulink, offering the possibility to simulate many times the ABS activity and to simulate the constructive parameters, road conditions or driver inputs changes, for a better optimization.

The mathematical model for the "quarter-car" and the adjusting of the wheel rolling during braking might be further developed for all four wheels, for the entire vehicle's system.

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